A Tale of Three LPAs: Some Notes on Zig-Zag Log-Periodic Arrays
2. The Trapezoidal Zig-Zag LPA

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Among the earliest 2-bay directional LPA designs was the trapezoidal zig-zag array. By 1960, Collins was producing a number of LPAs under the "327" designation, and an 11-60-MHz trapezoidal array with self-supporting tubular elements proved to be perhaps the most photogenic. The trapezoidal design also proved suitable for television reception with a design range of 48 to 230 MHz. In fact, the operating range (50-200 MHz) that we use in our modeling comparisons derives from the television LPDA, which radio amateurs adapted to use on 6, 4, 2, and 1.25 meter bands, depending upon one's location.

In this part of our exploration of the zig-zag LPA, we shall examine trapezoidal LPAs, beginning with those that employ a boom. **Fig. 1** shows the outline of a single trapezoidal bay, a 2-bay version, and side views of a range of versions showing different ψ-angles. All of the model outlines that appear in the figure employ a boom, the preferred configuration for the trapezoidal LPA. However, boomless versions are also theoretically possible, so we shall examine them also. Before we conclude this part, we shall also look at an older design from amateur literature, one that uses quite different values for α, τ, and ψ than we are employing in our attempt at a fair comparison among LPA varieties.

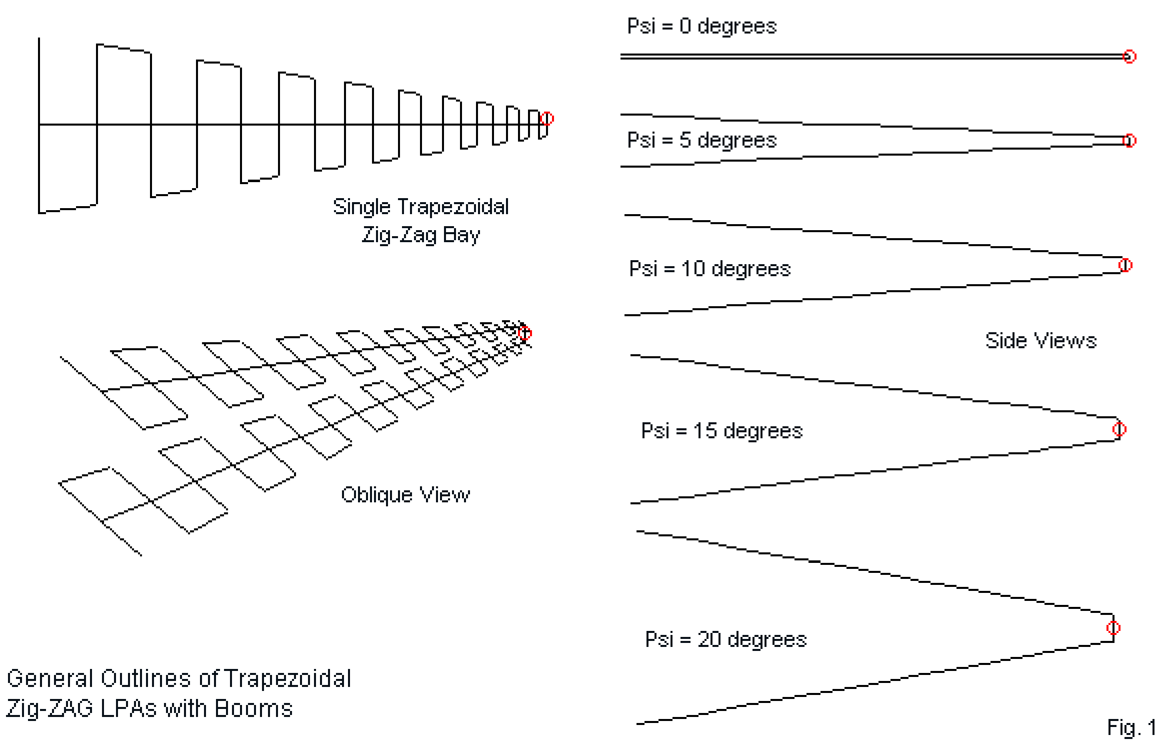


Fig. 1

Early zig-zag LPAs generally used quite large values for α, mostly for the practically reason of arriving at relatively short array boom lengths. 60° was perhaps the most common angle to appear in literature and commercial designs. Arrays also used τ-values between 0.6 and 0.8 to reduce the element count and arrive at a lighter overall structure. (Most LPDA literature recommends no τ-value below about 0.8, with higher values necessary to reduce the potential for the appearance of anomalous frequencies.) The most common ψ-angles seem to have been 35° to 45°.

Our comparison designs all use the following values: α’ = 17°, τ = 0.9, σ = 0.167, the optimum value for the selected value for τ. Therefore, the trapezoidal LPAs that we explore for most of our work in this part will use the dimensions shown in **Fig. 2**. The trapezoidal element lengths—counting only from side-to-side—will be identical to those used in Part 1’s LPDA. As well, the models will use 20 elements per bay. All models will be in free space and use 0.1” (2.54-mm) lossless wire for reasons outlined in Part 1.

Zig-Zag Log-Periodic Antenna Elements											Fig. 2
Work Sheet		**Bold = User Entry**									
Tau		**0.90**								Sigma	0.167
Alpha'	degrees	**17.00**	degrees	0.297	radians	1/2Alpha'	0.148	tanAlpha	0.1495		0.167
F-low		**50.00**	MHz								
F-high		**200.00**	MHz								
L-long		3.00	meters	9.84	feet	118.11	inches				
Lhigh		0.75	meters	2.46	feet	29.53	inches				
L*1.6		0.47	meters	1.54	feet	18.45	inches				
Rv	Vertex R	10.04	meters	32.93	feet	395.15	inches				
Element	Ln	Ln/2	Rn	Element	Lfeet	Lft/2	Rfeet	Element	Linch	Lin/2	Rinch
1	3.00	1.50	10.04	1	9.84	4.92	32.93	1	118.11	59.06	395.15
2	2.70	1.35	9.03	2	8.86	4.43	29.64	2	106.30	53.15	355.63
3	2.43	1.22	8.13	3	7.97	3.99	26.67	3	95.67	47.83	320.07
4	2.19	1.09	7.32	4	7.18	3.59	24.01	4	86.10	43.05	288.06
5	1.97	0.98	6.59	5	6.46	3.23	21.60	5	77.49	38.75	259.26
6	1.77	0.89	5.93	6	5.81	2.91	19.44	6	69.74	34.87	233.33
7	1.59	0.80	5.33	7	5.23	2.62	17.50	7	62.77	31.38	210.00
8	1.43	0.72	4.80	8	4.71	2.35	15.75	8	56.49	28.25	189.00
9	1.29	0.65	4.32	9	4.24	2.12	14.17	9	50.84	25.42	170.10
10	1.16	0.58	3.89	10	3.81	1.91	12.76	10	45.76	22.88	153.09
11	1.05	0.52	3.50	11	3.43	1.72	11.48	11	41.18	20.59	137.78
12	0.94	0.47	3.15	12	3.09	1.54	10.33	12	37.06	18.53	124.00
13	0.85	0.42	2.83	13	2.78	1.39	9.30	13	33.36	16.68	111.60
14	0.76	0.38	2.55	14	2.50	1.25	8.37	14	30.02	15.01	100.44
15	0.69	0.34	2.30	15	2.25	1.13	7.53	15	27.02	13.51	90.40
16	0.62	0.31	2.07	16	2.03	1.01	6.78	16	24.32	12.16	81.36
17	0.56	0.28	1.86	17	1.82	0.91	6.10	17	21.89	10.94	73.22
18	0.50	0.25	1.67	18	1.64	0.82	5.49	18	19.70	9.85	65.90
19	0.45	0.23	1.51	19	1.48	0.74	4.94	19	17.73	8.86	59.31
20	0.41	0.20	1.36	20	1.33	0.66	4.45	20	15.95	7.98	53.38

NEC-4 models of the trapezoidal zig-zag LPA are necessarily larger than comparable LPDA 2-bay models, with some models exceeding 1600 segments. Trapezoids require wires for both the boom (where used) and the wires that connect alternating element ends. The goal, within the limits of the segmentation process is to have all segments be the same length throughout the model. In addition, the segment length should also be within NEC limits at both the upper and the lower end of the operating range.

A number of published designs show the most forward element end and the boom brought to the vertex as a feedpoint location. However, considerable experimentation with variations showed no significant difference in performance in any category between this complex system and the simpler version used in the models. Models with booms add a wire that connects the two booms at the short end of the array and places the model source at the center of this wire. Models without booms add a wire connecting the free ends of the most forward element in each bay, with the source at its center.

Trapezoidal LPAs with Booms

Trapezoidal LPAs with booms derive most clearly from early versions of log-periodic structures using solid elements instead of the wire outline of those elements. We can easily picture these bays by using **Fig. 1** and simply filling in the area created by each closed trapezoidal structure. Experimental designs also tended to use a boom that became wider as the elements became longer, but practicalities reduced the boom to a constant diameter. In commercial and amateur versions of the antenna, the boom often used a larger diameter than the elements to provide structural support.

Trapezoids with booms show the general ability of a wire-outline version of the LPA to mimic the behavior of their solid surface counterparts. **Fig. 3** shows the 50-MHz current magnitude distribution along the wires of an array. The lower section of the graphic reveals the parallel current magnitude curves for each closed trapezoidal structure—at least for the longest elements that allow visual resolution of the curves.

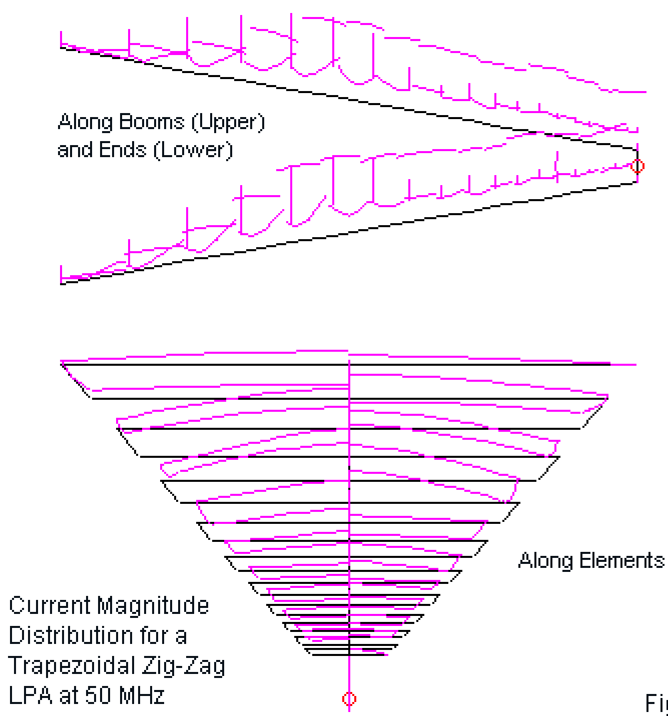


Fig. 3

The upper portion of the figure is equally significant, but for a different reason. The LPDAs shown in Part 1 presented the highest current magnitudes on the elements of the array closest to self-resonance at the operating frequency. Hence, the highest current at 50 MHz occurred on the second or third element from the rear of the array. In **Fig. 3**, the vertical lines represent peak relative element current, while the horizontal lines portray the current along the booms (upper lines) and the wire ends that close trapezoids (lower lines). Allowing for the sloping base lines of each bay, the peak current at the lowest operating frequency occurs well forward of the elements that are closest to self resonance. A similar situation will appear in boomless versions of the zig-zag trapezoidal LPA and in X versions of the antenna.

2-bay LPAs, with each bay transposing the closed trapezoids, are necessary to produce directional properties. It is tempting to think of the booms as constituting a phase line comparable to the line used to connect the dipoles of an LPDA. However, whenever we create a ψ-angle of any significant proportions, the spacing between the booms is too great for the structure to exhibit transmission-line properties. Indeed, the elements in each bay, with alternating directions, are capable of showing relatively correct relationships to each other at any given point along the length of the LPA, even without a boom. As we shall see, however, the presence of the boom enhances performance considerably by replicating in a wire-outline LPA the properties of the solid-surface LPA designs from which it derives.

One reason for including a version of the array using a ψ-angle of 0° is to allow us to see the change in the reference feedpoint impedance as the ψ-angle increases and as any transmission-line properties between booms disappear. We can see this and other changes by surveying sample performance properties of the five versions of the array at 50, 100, 150, and 200 MHz. **Table 1** provides the results in tabular form. Note initially the change in the reference impedance for the reported SWR values between ψ-angles of 5° and 10°.

Table 1. Sample performance values of trapezoidal zig-zag LPAs using various ψ-angles

1. 20 elements/bay, α = 17°, τ = 0.9, ψ = 0° (flat array, 4" separation between bays)

Frequency MHz	Max. Gain dBi	Front-Back Ratio dB	E BW degrees	H BW degrees	Impedance R +/- jX Ω	225-Ω SWR
50	7.17	27.83	70.6	113.8	240 - j 9	1.08
100	7.58	28.60	68.8	106.5	242 - j 2	1.08
150	7.61	24.91	67.8	107.4	222 - j 13	1.06
200	7.24	20.60	71.4	111.4	149 + j 28	1.55

2. 20 elements/bay, α = 17°, τ = 0.9, ψ = 5°

Frequency MHz	Max. Gain dBi	Front-Back Ratio dB	E BW degrees	H BW degrees	Impedance R +/- jX Ω	225-Ω SWR
50	8.34	30.50	65.1	94.4	261 - j 13	1.17
100	8.43	29.21	65.2	94.2	239 - j 6	1.07
150	8.20	25.16	65.4	97.4	225 - j 0	1.00
200	7.71	20.13	68.8	103.4	149 + j 39	1.59

3. 20 elements/bay, α = 17°, τ = 0.9, ψ = 10°

Frequency MHz	Max. Gain dBi	Front-Back Ratio dB	E BW degrees	H BW degrees	Impedance R +/- jX Ω	300-Ω SWR
50	9.16	26.94	64.0	82.8	293 + j 8	1.04
100	9.20	24.47	64.4	82.9	272 + j 33	1.17
150	8.90	20.91	65.0	85.6	259 + j 86	1.41
200	8.33	16.04	68.2	88.8	178 + j159	2.32

4. 20 elements/bay, α = 17°, τ = 0.9, ψ = 15°

Frequency MHz	Max. Gain dBi	Front-Back Ratio dB	E BW degrees	H BW degrees	Impedance R +/- jX Ω	300-Ω SWR
50	9.63	22.63	64.8	71.2	321 + j 17	1.09
100	9.51	21.90	65.3	72.4	314 + j 78	1.29
150	9.19	18.57	66.6	73.6	306 + j173	1.76
200	8.52	13.71	69.0	74.4	241 + j307	3.02

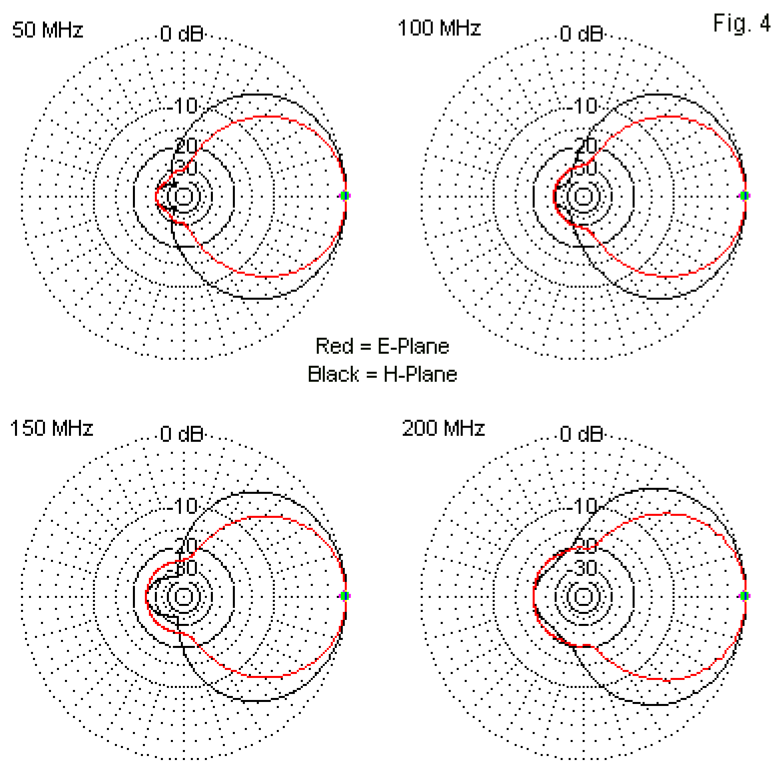
5. 20 elements/bay, α = 17°, τ = 0.9, ψ = 20°

Frequency MHz	Max. Gain dBi	Front-Back Ratio dB	E BW degrees	H BW degrees	Impedance R +/- jX Ω	300-Ω SWR
50	9.86	16.26	67.2	60.8	326 + j 10	1.09
100	9.68	18.87	67.1	61.1	351 + j 96	1.40
150	9.16	17.00	72.4	61.8	348 + j244	2.12
200	8.59	12.56	72.0	62.4	345 + j460	3.80

The gain figures are especially interesting, since the only difference among the arrays is the ψ-angle. At the lowest values of ψ, the gain at 50 MHz is significantly lower than the gain at 100 MHz. However, in the move between ψ=10° and ψ=15°, the 50-MHz gain rises to exceed the gain at 100 MHz. For all values of ψ, the gain at 200 MHz drops relative to the gain at the two middle frequencies in the sample. The decrease suggests that the design would benefit from additional shorter elements at the forward end of the array—assuming that equal gain across the operating passband is a desirable feature.

These characteristics are reflected in the impedance and SWR performance values. As we increase the ψ-angle, the higher frequencies show increased reactance as measured against the resistive component of the feedpoint impedance. The increasing reactive component also shows up as ever-worsening high-end SWR values relative to the indicated reference impedance.

Like all LPAs, increasing the ψ-angle produces increasing forward gain and decreasing front-to-back ratio values.



20 Elements/Bay, Alpha = 17°, Tau = 0.9, Psi = 5°
Sample Free-Space E-Plane and H-plane Patterns

Deciding upon a representative version of the array to provide graphical samples of performance is not easy for trapezoidal LPAs that have booms. Smaller values of ψ show better impedance and front-to-back behavior, while larger ψ-angle values show higher gain. In many ways, the 5° ψ-angle is a compromise. Still, as revealed in **Fig. 4**, the sample patterns for the selected version of the array show excellent pattern control at all frequencies. The H-plane pattern has a wider beamwidth than the E-plane pattern at this ψ angle. In fact, the E-plane and H-plane values to not merge until the ψ-angle reaches about 20°. (The data and graphics for all 5 versions of the trapezoidal LPA with a boom appear in the special data appendix to this series.)

The alternative method of data presentation that we used with the single- and double-bay LPDAs is also applicable to our trapezoidal LPAs. **Table 2** provides succinct summaries of some of the key sweep information for all 5 models.

Table 2. Frequency sweep summery of trapezoidal zig-zag LPAs using various ψ-angles from 50 to 200 MHz

1. ψ = 0°

Category	Minimum	Maximum	Δ	Average
Gain dBi	7.11	8.10	0.99	7.60
Front-Back dB	20.60	28.60	8.00	25.70
E Beamwidth °	65.0	71.6	6.6	68.4

2. ψ = 5°

Category	Minimum	Maximum	Δ	Average
Gain dBi	7.71	8.57	0.86*	8.35
Front-Back dB	20.13	32.04	11.91	27.12
E Beamwidth °	63.0	68.8	5.8	65.0

*Least variation across the passband of the group

3. ψ = 10°

Category	Minimum	Maximum	Δ	Average
Gain dBi	8.33	9.29	0.96	9.07
Front-Back dB	16.04	27.52	11.48	22.63
E Beamwidth °	62.7	68.2	5.5*	64.2

*Least variation across the passband of the group

4. ψ = 15°

Category	Minimum	Maximum	Δ	Average
Gain dBi	8.52	9.65	1.13	9.34
Front-Back dB	13.71	24.29	10.58	19.79
E Beamwidth °	63.2	69.0	5.8	65.4

5. ψ = 20°

Category	Minimum	Maximum	Δ	Average
Gain dBi	8.59	9.91	1.32	9.46
Front-Back dB	12.56	19.98	7.42	16.79
E Beamwidth °	63.4	72.4	9.0	67.2

The selected 5° ψ-angle shows at least 20-dB front-to-back ratio everywhere in the operating range. As the sample patterns show, the 180° value is also the worst-case value for this array. The average front-to-back value for the sweep decreases steadily with an increasing ψ-angle value, while the average gain-value increases. The E-plane beamwidth values remain relatively stable for all versions of the array. Indeed, the notations relating to the least variation in a performance category are not very significant, since the array show relatively even behavior in almost all versions. Except for the rapid decline in gain at the high end of the passband, the sweep graph for both gain and front-to-back ratio in **Fig. 5** for the 5° ψ-angle version reflects smooth performance with only the normal fluctuations created by the periodic structure. (Engineers already knew much of this behavior in the very late 1970s, as soon as method-of-moments analysis became available.)

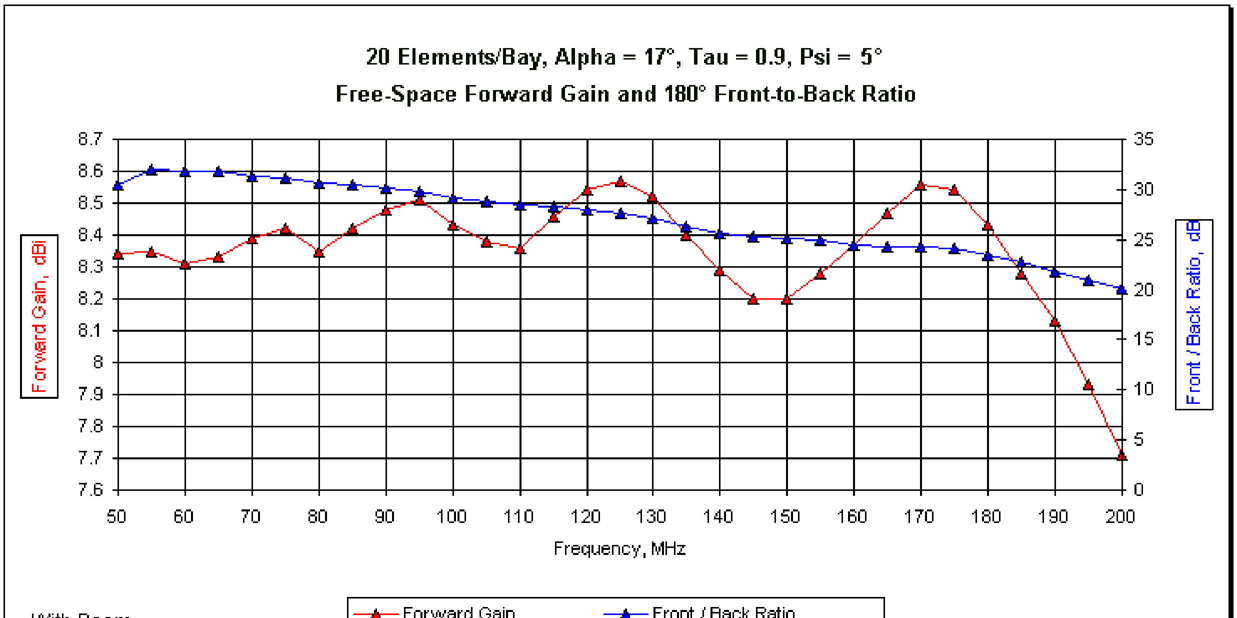


Fig. 5

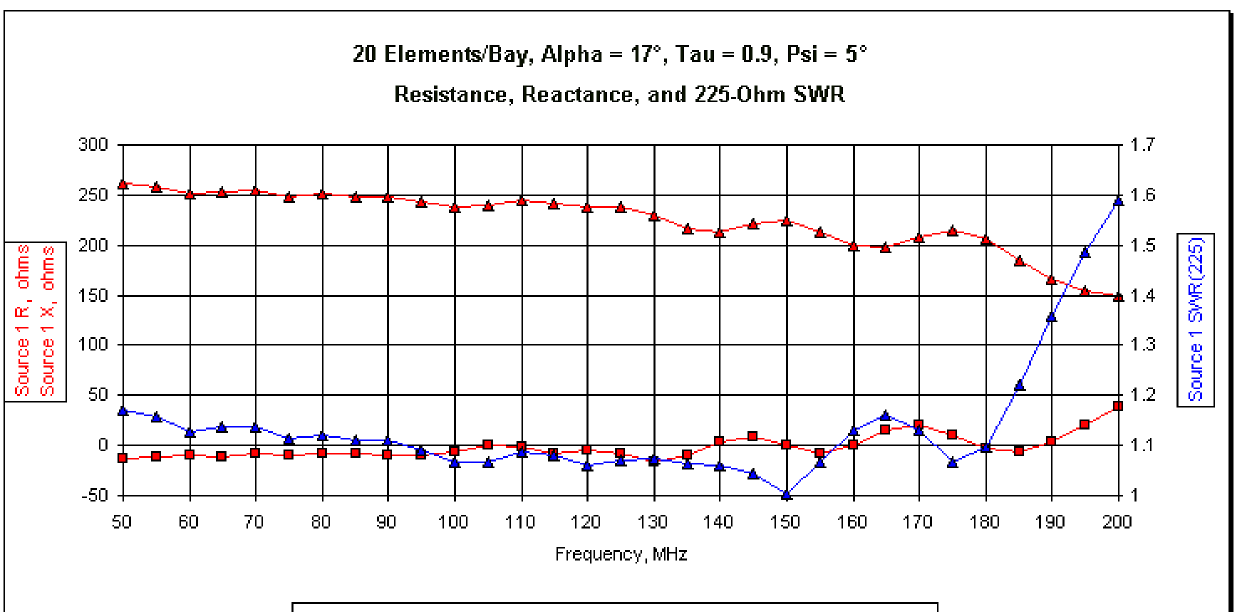


Fig. 6

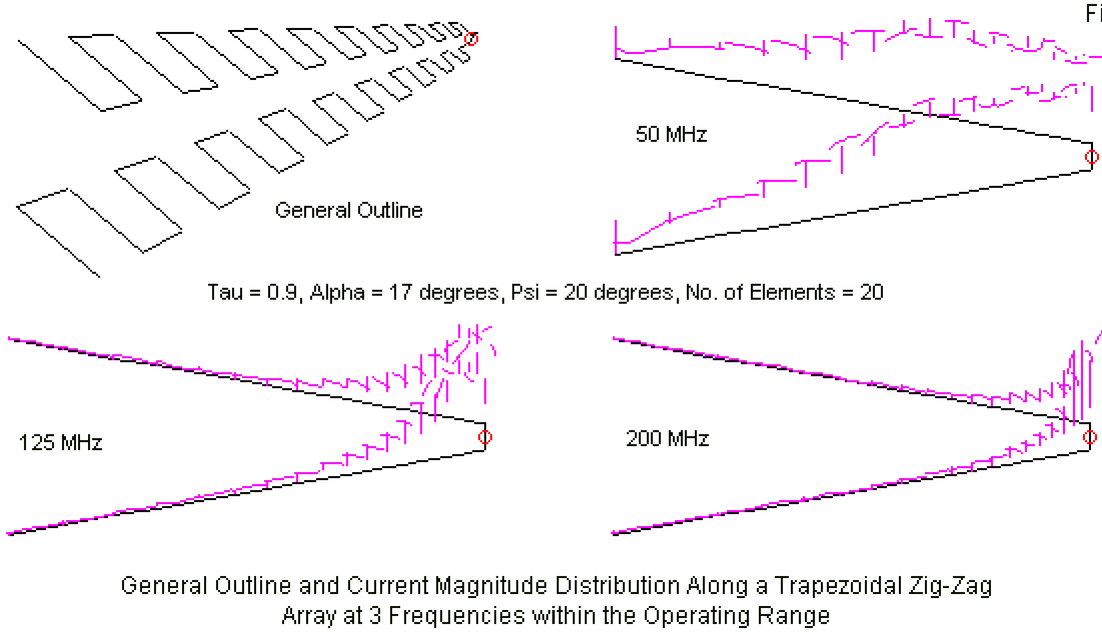
Fig. 6 provides sweep information on the free-space feedpoint performance in terms of resistance, reactance, and the 225-Ω SWR values. Performance is exceptionally smooth, with only the final 20 MHz of the passband showing the somewhat aberrant rise in the SWR value. As the graph shows, the rising SWR results from a combination of a sinking value for resistance and a slowly rising value of inductive reactance. As we increase the ψ-angle, the impedance curves become sharper with more sudden swings in value and especially a much greater rise in the feedpoint's inductive reactance. See the data appendix for details on the sweeps for all versions of the boom-equipped trapezoidal LPA.

Since the comparisons that we are making lack the imposition of outside construction or applications specifications, I have generally selected the representative example based on the smoothness of performance across the operating passband. External criteria might easily alter the selection for the design. In general, the representative trapezoidal LPA shows performance about mid way between the single- and double-bay LPDAs, although one may alter that position by a judicious selection of the ψ-angle. With a τ of 0.9, an α' of 17°, and a σ of 0.167, the boom-equipped trapezoidal LPA shows a high level of performance. It simplifies construction by not requiring a phase line, but in return, it demands a 3-dimensional structure.

Trapezoidal LPAs without Booms

Although it is theoretically possible to construct boomless trapezoidal LPAs, they are far less common than boomless LPAs using the X configuration. Both types of antenna offer some simplifications of a wire structure. One could build wire or thin-tube UHF versions of the array using a jig composed of a large board and nails to form corners with very small bending radii. **Fig. 7** shows the outline of such an array for 50-200 MHz, which uses the following values: τ = 0.9, α' = 17°, and ψ = 20°. The array uses 20 elements, all 0.1" in diameter, with the longest being a physical half-wavelength at the lowest operating frequency and the longest slightly shorter than a half-wavelength at 1.6 times the highest operating frequency. The dimensions are those in **Fig. 2**.

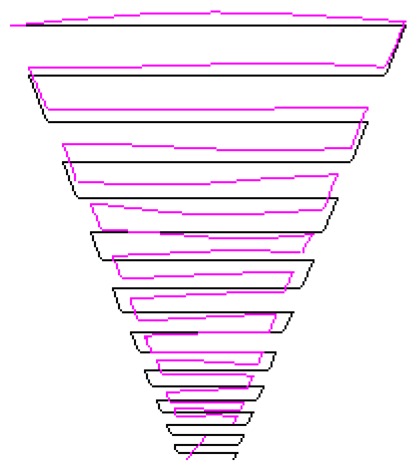
Fig. 7



General Outline and Current Magnitude Distribution Along a Trapezoidal Zig-Zag Array at 3 Frequencies within the Operating Range

Obviously, I consider boomless trapezoidal LPAs worth exploration, if only to compare the results with the data collected on trapezoidal zig-zag LPAs with booms. In addition to the outline, the figure contains current magnitude distribution graphs for 50, 125, and 200 MHz. The curves are somewhat difficult to interpret for a trapezoidal zig-zag array because the lines include current on the parallel elements (the vertical lines) and on the element-end linking wires (more horizontal lines). The element currents show the peak value of the current, which may occur anywhere along the element. Perhaps the most interesting fact about the curves is that they do not show maximum values very close to the element that is closest to resonance, as they do in comparable graphs for LPDAs. Instead, the peak current occurs generally much farther forward along the array. In this respect, they share a common general feature with boom-equipped versions of the array.

Fig. 8



50-MHz Current Magnitude Distribution
Along One Element Bay of a Trapezoidal
Zig-Zag LPA

Part of the reason for the seemingly displaced current maximum that is most apparent at the low end of the operating range results from the absence of the end-effect on the elements. Only the longest element in each bay has a free end. The end-wires have significant current magnitude and play a role in the transitions from one element to the next. As shown in the isolated bay curves in **Fig. 8**, mid-element current magnitude on some of the middle-length element may actually reach a minimum. Unlike the boom-equipped versions of the trapezoidal LPA, the boomless versions show no signs of correspondence to solid-surface LPAs that preceded them.

I modeled 4 versions of the 20-element trapezoidal zig-zag LPA, varying the value of ψ in 5° increments from 10° to 25°. One might experimentally replicate the exercise by using a variable angular brace in the general support structure for each bay. The volume of data collected is too great for inclusion in these notes. Therefore, I have made available a final document to serve as a data appendix for the complete data set. Here, we shall sample the data in two ways. The first is a table of sample performance information for 50, 100, 150, and 200 MHz for each of the 4 versions of the array. See **Table 3**. The most immediately striking fact is that the reference feedpoint impedance for the boomless trapezoids is about twice the value for version that use booms. Besides the difference in the reference impedance, the SWR value progression for each array resembles the progressions for versions with booms for ψ-angles between 10° and 20°. However, the cause of the rise is different. With boom-equipped arrays, the chief source of higher SWR values was an increase in reactance. With the boomless versions, the chief cause is a rapid rise in the feedpoint resistance.

Table 3. Sample performance values: trapezoidal array: 20 elements/bay, α = 17°, τ = 0.9

1. ψ = 10°

Frequency MHz	Max. Gain dBi	Front-Back Ratio dB	E BW degrees	H BW degrees	Impedance R +/- jX Ω	600-Ω SWR
50	6.20	9.85	101.1	102.4	566 + j 79	1.16
100	6.49	16.31	95.8	104.6	562 + j 99	1.20
150	6.26	16.26	101.6	110.2	521 + j 37	1.17
200	3.05	7.27	182.0	173.8	723 + j595	2.44

2. ψ = 15°

Frequency MHz	Max. Gain dBi	Front-Back Ratio dB	E BW degrees	H BW degrees	Impedance R +/- jX Ω	600-Ω SWR
50	6.34	8.94	107.2	94.6	530 + j 13	1.14
100	6.88	16.55	94.5	90.6	534 + j100	1.24
150	6.56	16.23	99.8	96.8	543 + j 65	1.16
200	3.62	5.95	179.4	129.4	958 + j596	2.43

3. ψ = 20°

Frequency MHz	Max. Gain dBi	Front-Back Ratio dB	E BW degrees	H BW degrees	Impedance R +/- jX Ω	600-Ω SWR
50	6.20	6.97	120.0	86.6	559 + j 1	1.07
100	7.02	10.51	96.2	76.6	550 + j121	1.26
150	6.63	13.57	103.7	86.2	594 + j 87	1.16
200	4.48	4.68	170.0	93.6	1378 + j345	2.47

4. ψ = 25°

Frequency MHz	Max. Gain dBi	Front-Back Ratio dB	E BW degrees	H BW degrees	Impedance R +/- jX Ω	600-Ω SWR
50	5.81	7.58	143.2	84.8	692 - j 57	1.18
100	7.43	6.87	89.2	61.8	590 + j139	1.26
150	6.67	8.20	108.8	69.4	645 + j 97	1.19
200	5.00	5.99	165.0	79.8	1325 - j327	2.37

The sample performance values clearly show the general trend toward higher gain and a lower front-to-back ratio as we increase the ψ-angle. The table does not include ψ-angles below 10° because the forward gain is too small to be useful. As the table shows, if we widen the ψ-angle, the gain performance at the lower end of the passband decreases, although it gradually improves at the higher end of the operating range. The chief limiting factor at the widest ψ-angle sampled is the rapid decline in the front-to-back ratio.

Selecting a representative version of the boomless trapezoidal LPA is difficult in the absence of external specifications dictated by an intended application. For this collection, I used an arbitrary but not irrelevant criterion: minimum variation of certain key values across the passband. Two of the obvious performance categories are variations in forward gain and variations in the E-plane beamwidth. We may gather the required information by performing frequency sweeps from 50 to 200 MHz in 5-MHz increments. **Table 4** provides a summary of the sweep data used in making the selection.

Table 4. Sweep data summary, 50-200 MHz in 5-MHz increments: trapezoidal array: 20 elements/bay, α = 17°, τ = 0.9

1. Ψ = 10°

Category	Minimum	Maximum	Δ	Average
Gain dBi	3.05	7.64	4.59	5.97
Front-Back dB	7.27	17.85	10.58	14.26
E Beamwidth °	76.8	182.0	105.2	108.1

2. Ψ = 15°

Category	Minimum	Maximum	Δ	Average
Gain dBi	3.62	7.93	4.31	6.31
Front-Back dB	5.95	17.75	11.80	14.14
E Beamwidth °	76.6	179.4	102.8	108.6

3. Ψ = 20°

Category	Minimum	Maximum	Δ	Average
Gain dBi	4.48	8.20	3.72*	6.57
Front-Back dB	4.46	13.60	9.14	9.94
E Beamwidth °	73.6	170.0	96.4*	109.7

*Least variation across the passband of the trapezoidal group

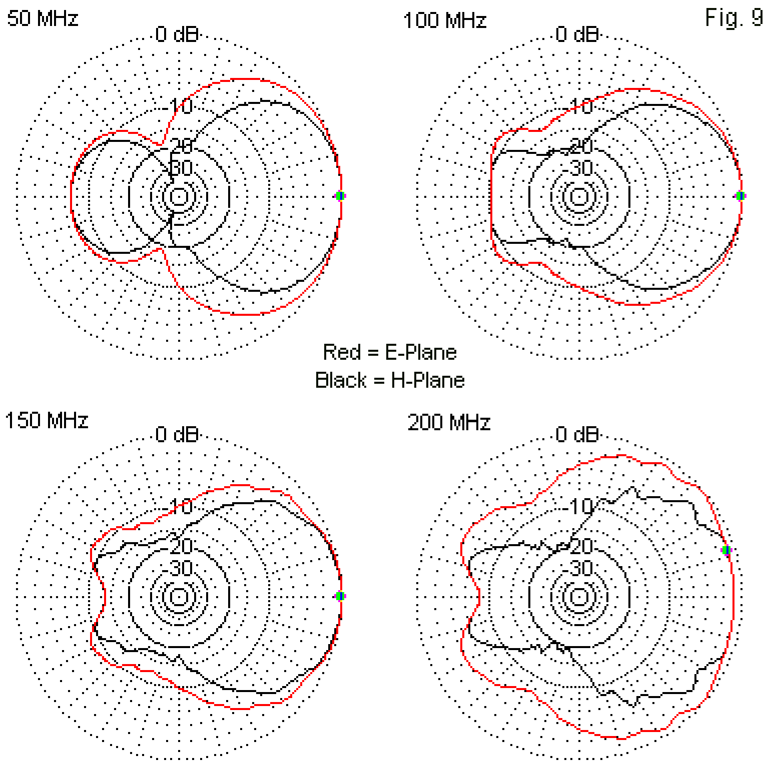
4. ψ = 25°

Category	Minimum	Maximum	Δ	Average
Gain dBi	4.84	8.89	4.05	6.79
Front-Back dB	3.97	8.68	4.71	6.52
E Beamwidth °	65.4	173.4	108.0	111.6

One reason for not including variations in the front-to-back ratio among the criteria for the selection of an optimal ψ-angle is a function of using the 180° front-to-back value. This value changes more radically from one frequency to the next, even though the quartering rear sidelobes may show a relatively constant worst-case front-to-back ratio. In addition, the front-to-back ratio shows the smallest range of change when its values are also the lowest and the least desirable generally.

One general feature of trapezoidal zig-zag arrays is a very wide average E-plane beamwidth at every value of ψ. The E-plane beamwidth of a 3-element Yagi, for example, is about 60°, while the lowest average E-plane beamwidth for the set of trapezoidal LPAs is 50° wider. In addition, all versions of the trapezoidal LPA show considerable beamwidth variation across the operating passband.

Based on the fact that the LPA with a 20° value for ψ shows the least variation in both gain and beamwidth, I have selected it as the optimal version for this exercise. Nonetheless, the average front-to-back ratio is under 10 dB, and the upper passband end design remains suspect. I did not try to correct the design, since the values for α and τ will also be used with the boomless X-version of the LPA. The 20° value for ψ also shows a weakness in the correlation between the E-plane and the H-plane beamwidths. Since the array consists of 2 bays having a constant distance (relative to element length) between elements in the H-plane, there is a ψ-angle at which the bays show nearly equal beamwidth values in both planes. For the trapezoidal LPA, that angle is about 10°. **Fig. 9** provides a gallery of overlaid E-plane and H-plane free-space patterns at ψ=20° to demonstrate by how much the selected value for ψ departs from equality. The data appendix will more clearly illustrate the rate of departure from equality of beamwidth as the value of ψ increases from 10° upward.



20 Elements/Bay, Alpha = 17°, Tau = 0.9, Psi = 20°
Sample Free-Space E-Plane and H-plane Patterns

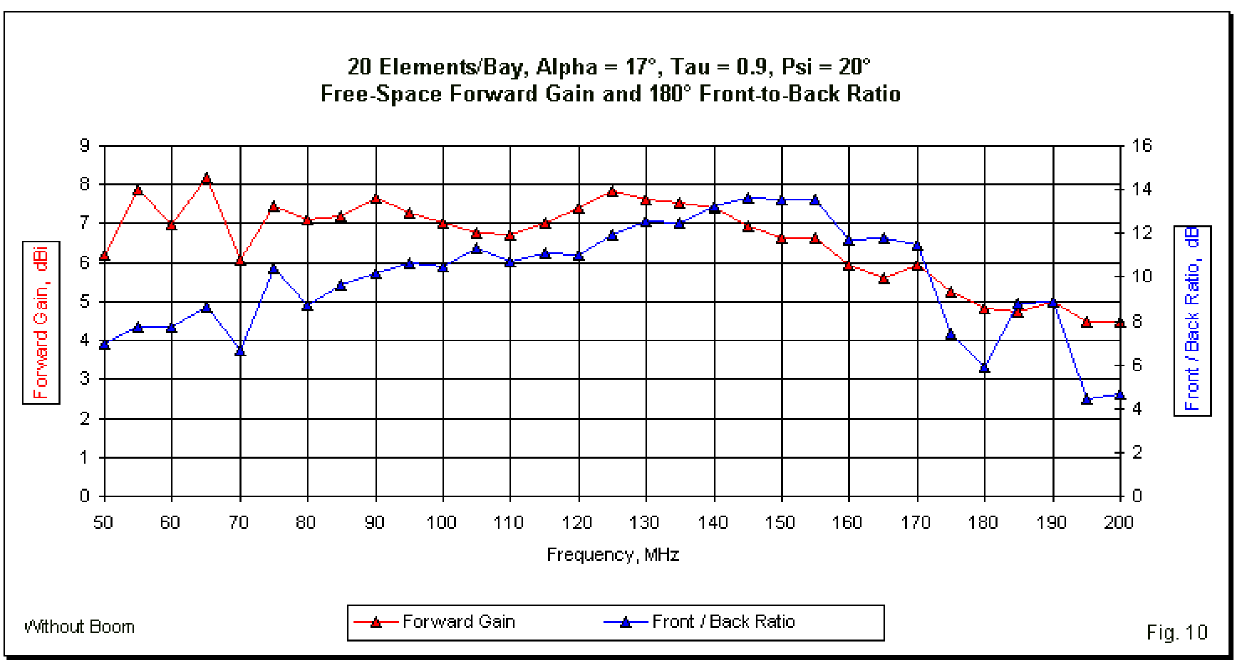


Fig. 10

Fig. 10 graphs the forward gain and the 180° front-to-back ratio across the passband. It shows a quite usable and relatively smooth set of values until the operating frequency exceeds about 175 MHz. The upper-end weakness and the need for redesign become apparent, especially from the front-to-back values. **Fig. 11** reinforces the design limitations by showing the rise in the feedpoint resistance and the 600-Ω SWR above the same break-point frequency. Since the degradation of performance affects virtually all categories of data in the upper 25 MHz of the design range, merely fussing with the feedpoint leads to improve the SWR value will not affect the basic capabilities of the trapezoidal LPA. Nothing short of adding elements toward the array vertex will do the job. We shall be interested in whether the X-version of the zig-zag LPA also requires such treatment.

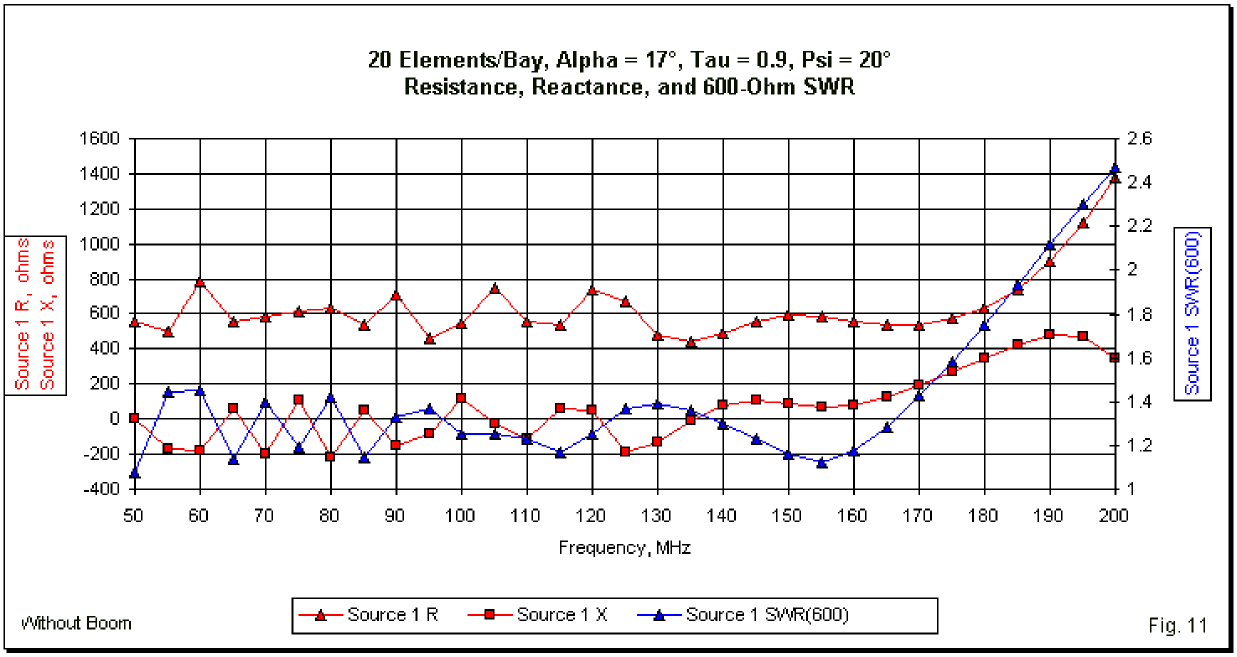


Fig. 11

The absence of a boom in this set of trapezoidal arrays results in inferior performance in several ways relative to corresponding versions equipped with booms. For ψ-angles between 10° and 20°, the boomless gain deficit is just about 3-dB on average. Moreover, the sweep graphs for the entire collection of boom-equipped arrays are considerably smoother than those for corresponding boomless arrays. Finally, the pattern smoothness of the boom-equipped trapezoidal LPA show better control than do the patterns for any of the boomless versions. The boomless trapezoidal zig-zag LPA seems hardly worth the effort of building, especially in a wire version in which one may easily add a boom wire.

A Trapezoidal LPA with Boom from Amateur Archives

Early designs of trapezoidal arrays attempted to cover the television frequencies from 48 to 230 MHz and to include coverage of the amateur bands within that range. (For U.S. television viewers, the range began at about 54 MHz because this country did not use channel 1. Until the perfection of cable systems, channel-2 TVI from 6-meter transmitting equipment remained a perennial problem.) The quest also included mechanical constraints in terms of boom length. Therefore, both amateur and commercial designs resorted to fairly low values of τ and correspondingly wide values of α'. Most of the designs from the 1960s have disappeared from available literature. However, we may find samples of zig-zag LPA designs in Chapter 26 of the

1995 edition of *Rothammels Antennebuch*, edited by Alois Krischke, DJ0TR. See section 26.6, “Logarithmisch periodische Antennen für VHF und UHF,” pp. 539-545. (Similar information appears beginning on p. 479 in the original edition of Karl Rothammel, Y21BK, *Antennenbuch* of 1984.) In this part of our work, we shall be most interested in the boom-equipped version that appears on pages 542-543 and is outlined in **Fig. 12**.

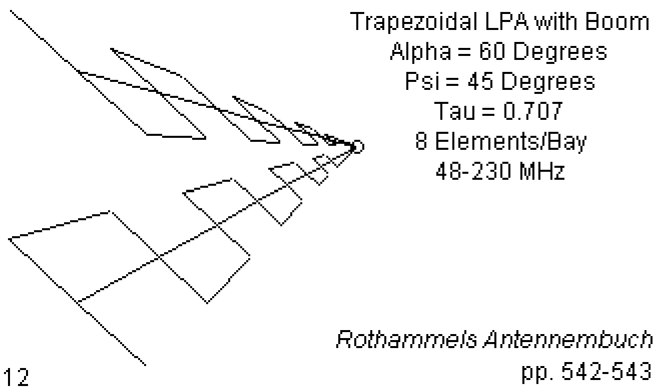


Fig. 12

Zig-Zag Log-Periodic Antenna Elements											Fig. 13
DARC Trapezoidal Design			p.542								
Work Sheet		**Bold = User Entry**									
Tau		**0.707**								Sigma	0.127
Alpha	degrees	**60.00**	degrees	1.047	radians	1/2Alpha	0.524	tanAlpha	0.5774		0.127
F-low		**50.00**	MHz								
F-high		**200.00**	MHz								
L-long		3.00	meters	9.84	feet	118.11	inches				
Lhigh		0.75	meters	2.46	feet	29.53	inches				
L*1.6		0.47	meters	1.54	feet	18.45	inches				
Rv	Vertex R	2.60	meters	8.52	feet	102.29	inches				
Element	Ln	Ln/2	Rn	Element	Lfeet	Lft/2	Rfeet	Element	Linch	Lin/2	Rinch
1	3.000	1.500	2.598	1	9.84	4.92	8.52	1	118.11	59.06	102.29
2	2.121	1.061	1.837	2	6.96	3.48	6.03	2	83.50	41.75	72.32
3	1.500	0.750	1.299	3	4.92	2.46	4.26	3	59.04	29.52	51.13
4	1.060	0.530	0.918	4	3.48	1.74	3.01	4	41.74	20.87	36.15
5	0.750	0.375	0.649	5	2.46	1.23	2.13	5	29.51	14.75	25.56
6	0.530	0.265	0.459	6	1.74	0.87	1.51	6	20.86	10.43	18.07
7	0.375	0.187	0.324	7	1.23	0.61	1.06	7	14.75	7.38	12.77
8	0.265	0.132	0.229	8	0.87	0.43	0.75	8	10.43	5.21	9.03

Fig. 13 provides the basic dimensions of the array. The calculated value of σ is very close to optimal for the selected value of τ. The shortest element is self-resonant at a frequency well above the “1.6F” specified in the basic calculations, suggesting but not guaranteeing good performance at the high end of the operating range. The short forward element has a second potential advantage: as shown in the outline, the element is close enough to the vertex of the array to permit the use of short boom and final wire sections that join there. The only modification of the original design that occurs in the model is the use of 0.1” (2.54-mm) lossless wire as the conductor.

The array—using the relatively thin wire—shows a natural feedpoint impedance close to 300 Ω. **Table 5** provides sample performance data for the free-space model at 50, 100, 150, and 200 MHz.

Table. 5. Sample performance values: 8 elements/bay, α = 60°, τ = 0.707, ψ = 45°

Frequency MHz	Max. Gain dBi	Front-Back Ratio dB	E BW degrees	H BW degrees	Impedance R +/- jX Ω	300-Ω SWR
50	6.65	9.11	75.2	99.0	322 – j182	1.79
100	6.11	9.38	75.2	111.8	302 - j 18	1.06
150	6.07	8.94	75.6	101.8	297 - j 14	1.05
200	7.02	9.01	63.0	79.6	286 + j 5	1.05

As expected, the array performance shows a bias toward the high end of the band. The low end of the operating range shows a perfectly usable but nevertheless high 300-Ω SWR value relative to the other frequencies. The gain and front-to-back values are roughly comparable to those we might obtain from a 2-element driver-reflector Yagi at each sampled frequency.

The relatively low forward gain and modest front-to-back ratios suggest that the patterns at the sampled frequencies might show considerably more variation from one frequency to the next when compared to the patterns in **Fig. 4** for the main model with a τ of 0.9. The samples of free-space E-plane and H-plane in **Fig. 14** confirm the suspicion, especially with respect to the H-plane patterns.

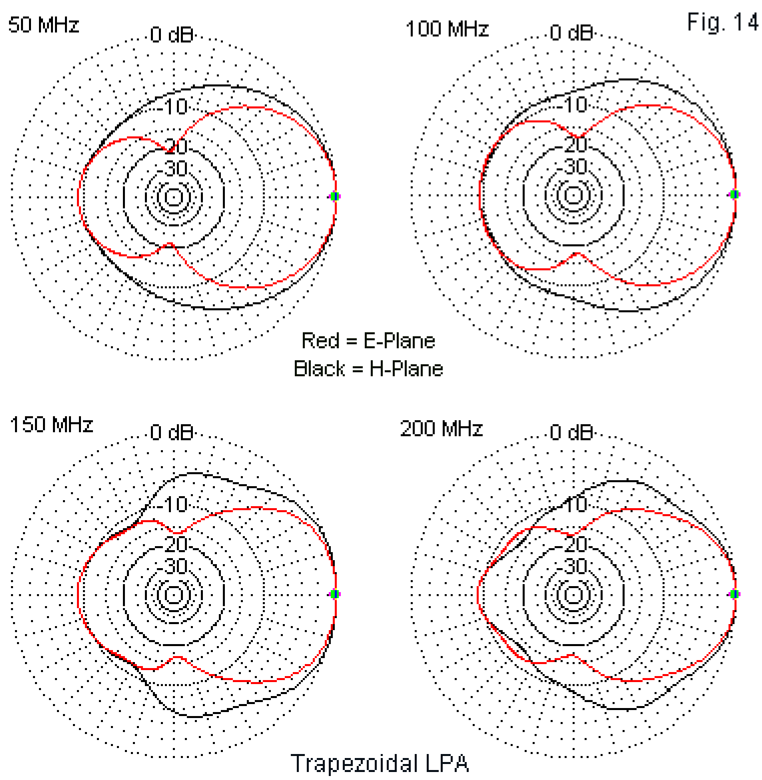


Fig. 14

Trapezoidal LPA
8 Elements/Bay, Alpha = 60 deg, Tau = 0.707, Psi = 45 Degrees
Sample Free-Space E-Plane and H-Plane Patterns

Table 6 summaries a few of the key performance categories across the operating range. Perhaps the only worrisome value in the table is the wide range of E-plane beamwidth values across the passband exhibited by the array. Despite the use of a low value of τ (0.707) and a wide value of α' (60°), the array show no anomalous frequencies at the 5-MHz increment for the sweep scans shown in **Fig. 15** and **Fig. 16**.

Table 6. Frequency sweep summary: 50-200 MHz: 8 elements/bay, α = 60°, τ = 0.707, ψ = 45°

Category	Minimum	Maximum	Δ	Average
Gain dBi	5.10	7.02	1.92	6.34
Front-Back dB	6.90	11.75	4.85	9.67
E Beamwidth °	54.6	96.0	41.4	74.1

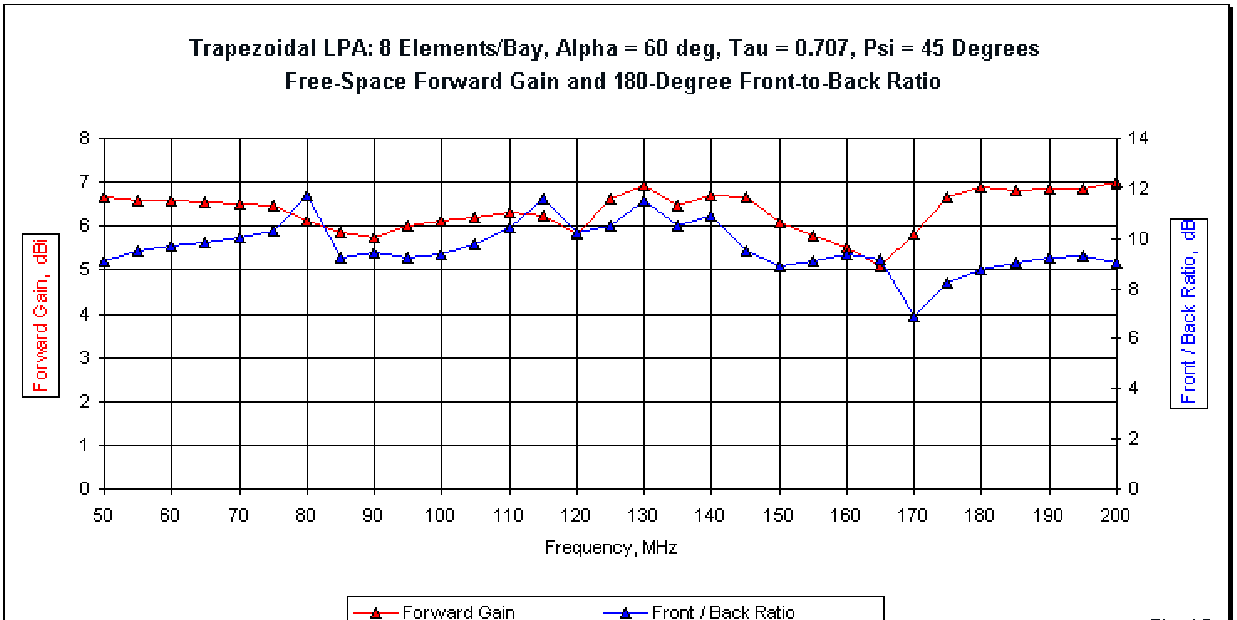


Fig. 15

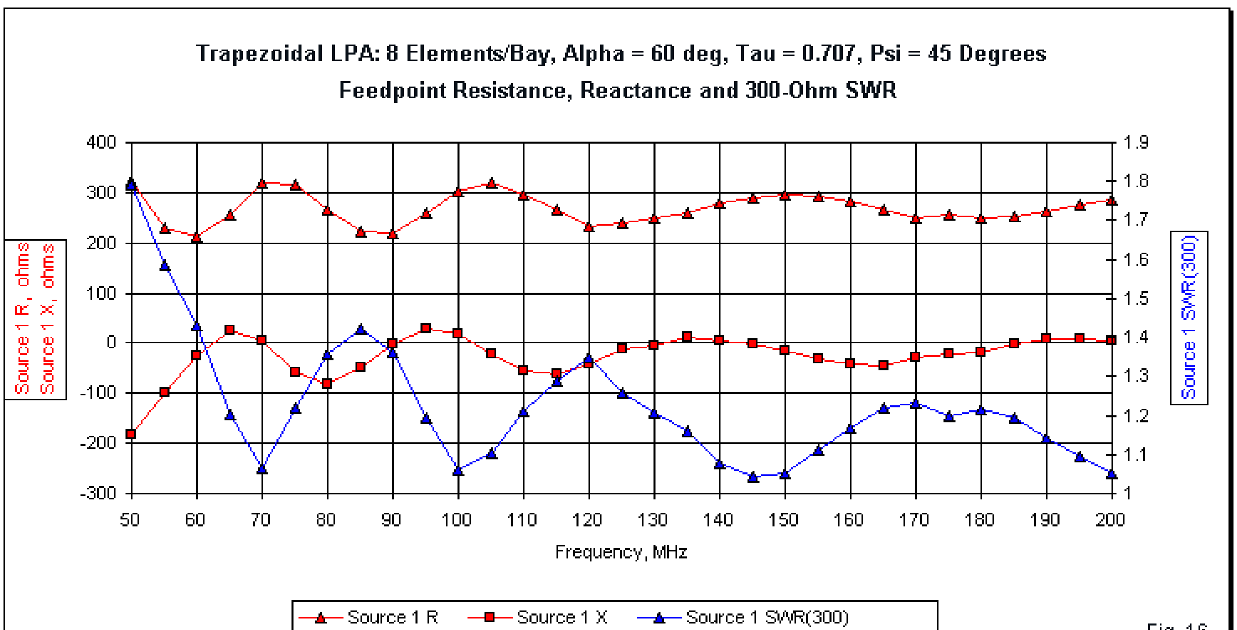


Fig. 16

One of the factors limiting array performance is the use of a very high value for ψ (45°). Early trapezoidal LPAs tended to use ψ-angles between 35° and 45° in order to assure the highest gain from the fewest elements. The present example is no exception. Without further study, one cannot say if the design has used too high a value for ψ and hence reached a point where the gain begins to drop. Apart from this open question, using a large ψ-angle to obtain maximum gain results in mediocre front-to-back performance and equally mediocre pattern control, as indicated by the range of E-plane beamwidth variation. Since the model uses relatively thin elements to avoid modeling pitfalls, these questions will remain open to further study.

I did experiment with incremental increases in the wire diameter. At a fairly low diameter (4-mm), the gain suddenly jumped by about 5 dB with an accompanying drop in the feedpoint impedance to values in the 50-75-Ω range. Checking the average gain test (AGT) values showed that the source of the jumps was elevated wire interpenetration at angular junctions. Corrected for the AGT value in each case, the array show an average gain rise of less than 0.1-dB per millimeter increase in wire diameter. Corrected feedpoint impedance reports showed a gradual decline from 300 Ω toward 200 Ω.

The end result is that the classic zig-zag trapezoidal LPA design that we have reviewed yields fairly mediocre performance, although the performance does cover the entire passband and may be acceptable for some applications. One may design the zig-zag array using wide values of α' and low values of τ without encountering the anomalous frequencies of reversed patterns that are common to single-bay LPDAs trying to use the same values.

Conclusion to Part 2

Our brief journey through trapezoidal zig-zag LPAs has produced some interesting results. Using the relatively high τ of 0.9 and the relatively narrow α' of 17°--with accompanying low values for ψ—the trapezoidal array equipped with a boom replicates many of the performance characteristics of it solid-surface counterpart. It is capable of exceeding the gain of a single-bay LPDA using the same design parameters, but falls short of a 2-bay LPDA. The front-to-back ratio is good, if we compare it to typical Yagi performance, but falls short of what we may obtain from an LPDA. The performance and impedance curves are generally quite smooth across the operating passband. In short, the boom-equipped trapezoidal LPA is a very serviceable array where the builder prefers not to wrestle with a phase line. In contrast, the theoretically possible boomless version of the trapezoidal array has significantly lesser performance that hardly merits construction.

Classic early trapezoidal zig-zag designs—with lower values of τ and much wider values of α'—lack a convenient summery evaluation. Although relatively modest in general performance, they can provide that performance over a wide frequency span and may prove to be adequate for some applications. Optimizing such designs within their avowed limitations in boom length is a task outside the scope of this study.

Theoretically, the X or saw tooth zig-zag LPA is the equivalent of the trapezoidal array. To what degree theory holds up in practice—using our standard design parameters—will be the subject of the final part in this series.